Electrodynamics and Optics Cheatsheet

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Basic relations

Maxwell

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho \qquad \mathbf{D} = \epsilon\epsilon_0 \mathbf{E}$$

$$\nabla \times \mathbf{E} = \dot{\mathbf{B}}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}} \qquad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} = \frac{\mathbf{B}}{\mu\mu_0}$$

Energy and impedance

$$\mathbf{N} = \mathbf{E} \times \mathbf{H}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\dot{\mathbf{A}} - \nabla\phi$$

$$c^2 = \frac{1}{\mu_0\epsilon_0}$$

$$\frac{|\mathbf{E}|}{|\mathbf{B}|} = \frac{\omega}{k} = v = \frac{c}{n} = \frac{c}{\sqrt{\epsilon\mu}}$$

$$\frac{|\mathbf{E}|}{|\mathbf{H}|} = \frac{c\mu\mu_0}{\sqrt{\epsilon\mu}} = \sqrt{\frac{\mu\mu_0}{\epsilon\epsilon_0}} = Z = \sqrt{\frac{\mu}{\epsilon}} Z_0$$

Boundary conditions

$$D_\perp = D_\perp$$

$$\theta_B = \arctan\left(\frac{n_2}{n_1}\right) \implies r_p = 0$$

Anisotropic

Uniaxial: $\epsilon_1 = \epsilon_2 \neq \epsilon_3$
perpendicular to $\hat{\mathbf{e}}_3 \implies$ ordinary spherical wavefront, $\mathbf{k} \parallel \mathbf{N}$;
not perpendicular to $\hat{\mathbf{e}}_3 \implies$ extraordinary wave.

Induced birefringence

chiral materials

$$n_L \neq n_R \implies \text{rotation}$$

$$\alpha = \frac{\pi}{\lambda}(n_L - n_R); \qquad \Delta\phi = 2\alpha d$$

Faraday effect

$$m\ddot{\mathbf{r}} = -e(\mathbf{E} + \dot{\mathbf{r}} \times \mathbf{B_0})$$

$$\theta = V B_0 d \qquad \text{N.B. directional}$$

Kerr effect

$$\Delta n = n_x - n_y = \lambda_0 k E_0^2$$

Incoherence

Degree of polarisation: $V = \frac{I_{\text{pol}}}{I_{\text{tot}}}$

Coherence

$$F = \frac{1}{\omega - \omega_0 - i\omega_s}$$

width

$$\Delta\omega = \text{FWHM} = 2\omega_s = 2.36\sigma$$

lifetime

$$\tau_s = \frac{1}{\omega_s}$$

thermal

$$\sigma = \omega_0 \sqrt{\frac{k_B T}{mc^2}}$$

pressure

$$\tau = \frac{1}{4N\bar{v}A}$$

Thermodynamic potentials

Gibbs-Duhem

$$U = TS - pV + \mu N$$

$$dU = T\,dS - p\,dV + \mu\,dN$$

$$d\mu = -s\,dT + v\,dp$$

Availability

$$\begin{aligned} dA &= dU - T_R\,dS + p_R\,dV - \mu_R\,dN \\ &= (T - T_R)\,dS - (p - p_R)\,dV + (\mu - \mu_R)\,dN \\ &= -T_R\,dS_{\text{tot}} \end{aligned}$$

$$dA \leq 0 \iff dS_{\text{tot}} \geq 0$$

$$H = U + pV$$

$$F = U - TS = -k_B T \ln Z$$

$$G = U - TS + pV = \mu N$$

$$\Phi = U - TS - \mu N = -pV = -k_B T \ln \Xi$$

$$(\mathrm{d})_{\text{natural}} = dA$$

$$\Xi = \sum e^{-\beta(E_i - \mu N_i)}$$

Statistical physics

Statistics

$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} - \eta}$$

Quantum

$$\lambda = \left(\frac{2\pi\hbar^2}{m k_B T} \right)^{\frac{1}{2}}$$

$$n_c = \frac{1}{\lambda^3}$$

$$Z_N = \frac{1}{N!} Z_1^N$$

$$Z = \frac{1}{N!} \left(\frac{V}{\lambda^3} \right)^N$$

Bose condensation

$$N = \int n(\epsilon) g(\epsilon) \,\mathrm{d}\epsilon$$

At some low, finite temperature, $\mu = 0$.

Virial expansion

$$\langle \nu \rangle = \left\langle -\frac{1}{2} \sum_i \mathbf{r}_i \cdot \mathbf{f}_i \right\rangle = \langle \nu_{\text{ext}} \rangle = \langle \nu_{\text{int}} \rangle$$

$$\langle \nu_i \rangle = \frac{1}{2} \int_0^\infty 4\pi r^2 n e^{-\beta\phi} r \frac{\mathrm{d}\phi}{\mathrm{d}r} \,\mathrm{d}r$$

$$\langle \nu \rangle = \frac{3}{2} k_B T$$

$$\frac{p}{k_B T} = n + B_2(T) n^2 + \dots$$

$$B_2(T) = \int_0^\infty 2\pi r^2 \left(1 - e^{-\beta\phi}\right) \mathrm{d}r$$

Equilibrium fluctuation

$$\left\langle x^2 \right\rangle = \frac{1}{Z\beta^2} \frac{\partial^2 Z}{\partial f^2} \qquad E = -fx$$

$$\left\langle \Delta M^2 \right\rangle = k_B T \left(\frac{\partial \langle M \rangle}{\partial B} \right)_T$$

$$\left\langle \Delta M^2 \right\rangle = k_B T^2 \frac{\partial U}{\partial T}$$

Fluctuation-dissipation theorem

$$\left\langle \Delta x^2 \right\rangle = \frac{k_B T_R}{\left(\frac{\partial^2 A}{\partial x^2} \right)_{x=x_0}} = \frac{k_B T}{\left(\frac{\partial f}{\partial x} \right)_{x=x_0}}$$