Quantum Condensed Matter Physics Cheatsheet

Feiyang

Conduction

Lorentz oscillator

$$m\ddot{x} = -m\omega_T^2 x - m\gamma\dot{u} + qE$$
$$p = \frac{N}{V}ex$$
$$\chi = \frac{P}{\epsilon_0 E}$$

Drude model

$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{\tau}\right)\mathbf{j} = \frac{N}{V}\frac{q^2}{m}\mathbf{E} + \frac{\mathbf{q}}{\mathbf{m}}(\mathbf{j}\times\mathbf{B})$$
$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_P^2}{\omega^2 + i\omega/\tau} \qquad \omega_P^2 = \frac{ne^2}{m\epsilon_0}$$

Hall effect

Hall coefficient

$$v_x = \frac{q\tau}{m}E_x$$
$$Bv_x = E_y$$
$$E_y = \frac{qB\tau}{m}E_x$$
$$j_x = nv_x q$$
$$R_H = \frac{E_y}{j_x B} = \frac{Bv_x}{j_x B} = \frac{1}{nq}$$

Thomas-Fermi screening

$$q_{TF}^2 = \frac{e^2 g_v(E_F)}{\epsilon_0}$$
$$n(q) = \frac{\epsilon_0 q^2}{e}\frac{V_{\text{ext}}(q)}{1 + q^2/q_{TF}^2}$$
$$n(r) = \frac{q_{TF}^2}{4\pi}\frac{Q}{e}\frac{e^{-q_{TF}r}}{r}$$

Lattice heat capacity

Debye model

$$\omega = vk$$
$$D_D(\omega) = \frac{4\pi k^2}{(2\pi/L)^3}\frac{\mathrm{d}k}{\mathrm{d}\omega} = \frac{V\omega^2}{2\pi^2 v^3}$$
$$N = \int_0^{\omega_D} D_D(\omega)\,\mathrm{d}\omega$$
$$U = \int \mathrm{d}\omega\, D(\omega) \underbrace{n(\omega)}_{\text{Bose-Einstein}} \hbar\omega$$
$$U \propto \beta^{-4} \qquad \text{for } \beta\hbar\omega_D \gg 1$$
$$C_V = qNk_B\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x\,\mathrm{d}x}{(e^x-1)^2}$$

electron gas

$$u = \int \mathrm{d}E\, E g(E)\ \text{Fermi-Dirac}(E)$$
$$c_V \approx \frac{N}{V}\frac{k_B^2}{E_F}T$$

Periodicity

Bragg condition: $\mathbf{q}\cdot\mathbf{R} = \mathbf{2\pi m} \iff \mathbf{q} = \mathbf{G}$

Bragg's law: $\mathbf{k}\cdot\mathbf{G/2} = \mathbf{(G/2)^2}$

Periodic potential: $\left[\hat{\mathbf{H}}, \hat{\mathbf{T}}\right] = \mathbf{0}$, $\hat{T}$ is a symmetry

Bloch's theorem

$$\psi_q(x) = e^{iqr}\underbrace{u_{j,q}(r)}_{\text{periodic}}$$

Bloch state construction

Nearly free electron model

$$|\psi_k\rangle = \sum c_{k-G}\,|k-G\rangle$$
$$H = \underbrace{T}_{\text{diagonal}} + \underbrace{V}_{\text{periodic, off-diagonal}}$$

Tight binding (LCAO)

one orbital per site, nearest neighbour only

$$H = T + V_a + V_b$$
$$(T+V_a)\,|a\rangle = |E_a\rangle$$
$$(T+V_b)\,|b\rangle = |E_b\rangle$$
$$H = \begin{pmatrix} E_a + \langle a|\,V_b\,|a\rangle & t \\ t^* & E_b + \langle b|\,V_a\,|b\rangle \end{pmatrix}$$
$$t = \langle a|\,(T + V_a + V_b)\,|b\rangle$$
$$E = \langle 0|\,\hat{H}\,|\psi\rangle = \sum e^{ikR_n}\langle 0|\,\hat{H}\,|n\rangle$$

Pseudopotential: smooth near the core, remove core (non-valence) e^-.

Bands

Density of states of n-th band

$$g_n(E) = \int \frac{\mathrm{d}S}{4\pi^3}\frac{1}{|\boldsymbol{\nabla}_\perp E_n(k)|}$$

holes

$$k_h = -k_e$$
$$\epsilon_h(k_h) = -\epsilon_e(k_e)$$
$$v_h = v_e$$
$$m_h^* = -m_e^*$$

photoemission

$$E_f = E_i + \hbar\omega - \phi \qquad k_{f\parallel} = k_{i\parallel}$$

Semiconductors

Law of mass action (intrinsic semiconductors)

$$|E-\mu| \gg k_B T$$
$$n = \int_{E_c}^{\infty} \mathrm{d}E\, g_e(E) f(E)$$
$$p = \int_0^{E_v} \mathrm{d}E\, g_h(E) f(E)$$
$$np = n_c n_v e^{-\overbrace{E_g}^{E_c - E_v}\beta}$$

doped e.g. with donors of binding energy just below conduction band by Δ_d

$$n = \sqrt{n_c N_d} e^{-\Delta_d \beta/2}$$

currents

$$\text{generation (minority)} \quad -J^{\text{gen}}$$
$$\text{recombination (majority)} \quad J^{\text{rec}} = e^{-\beta e(\phi_b - V)} n_i^2$$
$$I = n_i^2 e^{-\beta \overbrace{e\phi_b}^{E_g}} \left(e^{\beta e V} - 1\right)$$

A *quantum well* traps carriers into bound states near a real space region to create lasers.

Magnetism

Currie

$$p(m) = \frac{e^{-\beta E(m)}}{Z}$$
$$m = \int m p(m)\, \mathrm{d}^2 m$$
$$\chi = \frac{\mathrm{d}m}{\mathrm{d}H} = \mu_0 \beta \langle m_i m_j \rangle \qquad \text{for } H = 0$$
$$m^2 = \frac{1}{3}\mu^2$$
$$\chi = \frac{1}{3}\frac{N}{V}\mu_0 m y^2 \beta$$

Heisenberg

$$\text{(anti)clustering} \implies H \text{ depends on } \mathbf{S} \implies \mathbf{S}_1 \cdot \mathbf{S}_2$$
$$H_2 = -2J\, \mathbf{S}_1 \cdot \mathbf{S}_2$$

Pauli

Add in penalty U for doubly-filled states

$$\epsilon_{k\uparrow} = \epsilon_k - \mu_B B_a + U n_\downarrow$$
$$\epsilon_{k\downarrow} = \epsilon_k + \mu_B B_a + U n_\uparrow$$
$$\mu + \mu_B B_a = E_{F\uparrow}$$
$$\mu - \mu_B B_a = E_{F\downarrow}$$
$$n_{\uparrow/\downarrow} = \int \mathrm{d}\epsilon\, \frac{1}{2} g(\epsilon) f(\epsilon) = \int_0^{\mu_{\uparrow/\downarrow}} \mathrm{d}\epsilon\, \frac{1}{2} g(\epsilon)$$
$$\frac{N}{V}(n_\uparrow - n_\downarrow) = \frac{1}{2} g(E_F) \int_{\mu_\downarrow}^{\mu_\uparrow} \mathrm{d}\epsilon$$
$$M = \mu_B (n_\uparrow - n_\downarrow) \implies \chi_\sigma = \mu_0 \frac{\mu_B^2 g(E_F)}{1 - \frac{U g(E_F)}{2}}$$
$$\frac{U g(E_F)}{2} > 1 \implies \text{Ferromagnetism}$$

Quantum oscillations

$$\oint p\, \mathrm{d}r = \left(n + \frac{1}{2}\right) h = q \oint (r \times B + A)\, \mathrm{d}r = -q\Phi$$
$$\Phi_n = A_r^{(n)} B_n = \left(n + \frac{1}{2}\right)\frac{h}{e}$$
$$A_k = \left(\frac{Bq}{\hbar}\right)^2 A_r = \frac{2\pi e}{\hbar} B \left(n + \frac{1}{2}\right)$$

Fermi liquid

$$\Gamma \sim \epsilon^2$$

Quasiparticles

$$\psi_k(\mathbf{r}, \mathbf{t}) = \psi_k(r) e^{-i\epsilon_k t/\hbar} e^{-\Gamma_k |t|}$$
$$A = -\frac{1}{\pi}\, \mathrm{Im}\left[\frac{1}{\omega - \epsilon_k/\hbar + i\Gamma_k}\right]$$

Heavy Fermions due to interaction

$$g(E_F) \text{ high} \implies \text{magnet}$$