Thermal and Statistical Physics Cheatsheet

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Thermodynamic potentials

Gibbs-Duhem

$$U = TS - pV + \mu N$$
$$\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V + \mu\,\mathrm{d}N$$
$$\mathrm{d}\mu = -s\,\mathrm{d}T + v\,\mathrm{d}p$$

Availability

$$\begin{aligned}\mathrm{d}A &= \mathrm{d}U - T_R\,\mathrm{d}S + p_R\,\mathrm{d}V - \mu_R\,\mathrm{d}N \\ &= (T - T_R)\,\mathrm{d}S - (p - p_R)\,\mathrm{d}V + (\mu - \mu_R)\,\mathrm{d}N \\ &= -T_R\,\mathrm{d}S_{\text{tot}}\end{aligned}$$
$$\mathrm{d}A \le 0 \iff \mathrm{d}S_{\text{tot}} \ge 0$$

$$H = U + pV$$
$$F = U - TS = -k_B T \ln Z$$
$$G = U - TS + pV = \mu N$$
$$\Phi = U - TS - \mu N = -pV = -k_B T \ln \Xi$$
$$(\mathrm{d})_{\text{natural}} = \mathrm{d}A$$
$$\Xi = \sum e^{-\beta(E_i - \mu N_i)}$$

Statistical physics

Statistics

$$\langle n_k \rangle = \frac{1}{e^{\beta(\epsilon_k - \mu)} - \eta}$$

Quantum

$$\lambda = \left(\frac{2\pi\hbar^2}{mk_B T}\right)^{\frac{1}{2}}$$
$$n_c = \frac{1}{\lambda^3}$$
$$Z_N = \frac{1}{N!} Z_1^N$$
$$Z = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N$$

Bose condensation

$$N = \int n(\epsilon) g(\epsilon)\,\mathrm{d}\epsilon$$

At some low, finite temperature, $\mu = 0$.

Virial expansion

$$\langle \nu \rangle = \left\langle -\frac{1}{2}\sum_i \mathbf{r}_i \cdot \mathbf{f}_i \right\rangle = \langle \nu_{\text{ext}} \rangle = \langle \nu_{\text{int}} \rangle$$
$$\langle \nu_i \rangle = \frac{1}{2}\int_0^\infty 4\pi r^2 n e^{-\beta\phi} r \frac{\mathrm{d}\phi}{\mathrm{d}r}\,\mathrm{d}r$$
$$\langle \nu \rangle = \frac{3}{2} k_B T$$
$$\frac{p}{k_B T} = n + B_2(T) n^2 + \dots$$
$$B_2(T) = \int_0^\infty 2\pi r^2 \left(1 - e^{-\beta\phi}\right)\mathrm{d}r$$

Equilibrium fluctuation

$$\left\langle x^2 \right\rangle = \frac{1}{Z\beta^2}\frac{\partial^2 Z}{\partial f^2} \qquad E = -fx$$
$$\left\langle \Delta M^2 \right\rangle = k_B T \left(\frac{\partial \langle M \rangle}{\partial B}\right)_T$$
$$\left\langle \Delta M^2 \right\rangle = k_B T^2 \frac{\partial U}{\partial T}$$

Fluctuation-dissipation theorem

$$\left\langle \Delta x^2 \right\rangle = \frac{k_B T_R}{\left(\frac{\partial^2 A}{\partial x^2}\right)_{x=x_0}} = \frac{k_B T}{\left(\frac{\partial f}{\partial x}\right)_{x=x_0}}$$